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A Numerical Treatment of a Mathematical Model of Ground Water Flow in Rice Field near Marine Shrimp Aquaculture Farm

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Abstract

The high level salinity in soil is a main problem of a plant grows less in rice field near marine shrimp aquaculture farm. The salinity in soil and ground water flow are required for soil quality assessments. The modeling often involves numerical methods to solve the equations. In this research, two mathematical models are used to simulate the salinity in the ground with varied flow velocity. The first is a potential flow model that provides the velocity field of the ground water flow. The second is a dispersion model, where the commonly used governing factor is the two-dimensional dispersion equation that gives the salinity fields. In the simulation process, we used the finite difference method for the both models. Finally, we present a numerical simulation that confirms the results of the technique.

Keywords: Salinity; Marine shrimp aquaculture farm; Rice field; Ground water flow; Potential flow; Advection-dispersion equation; Finite difference method

1. Introduction

The saline water from the sea is filled into the shrimp aquaculture farm. The saline water is the salty water that disperses to the ground as shown in Fig. 1. The soil in the agricultural area near shrimp aquaculture farm will becomes high level of salinity. Salty soils are a common feature and an environmental problem in irrigated lands in arid and semi-arid regions. They have poor or little crop production [5]. The indicator that can be figure out the salinity level of soil is its electrical conductivity.

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The electrical conductivity profile of soil in each area can be measured by the methods of field measurement and mathematical modeling. For the field measurement, SYSCAL KID is important equipment for electrical conductivity profile monitoring. Although, the field measurement is not suitable for rice field near shrimp aquaculture farm that mostly wet area.

On the other hand, the mathematical model is practical for the problem. The salt concentration was strongly correlated with the conductivity of the ground in [6]. In [2], the conductivity of the ground is used to indicate the amount of salt and conductive mineral deposit in the ground. There are many model can be describe the problem. In [1], they describe results from a transient electromagnetic (TEM) system that has been deployed to monitor the influx of saline water through sub-riverbed sediment. The direct current method that can be describing the conductivity of soil is used in [4]. In [4], they give the method of measurement the salinity of soil on the area near shrimp aquaculture farm in Nakornpathom province, Thailand. The standard soil has electric conductivity lower than 0.4 Siemens/meter that suitable for rice plantation.

In [7], they modified the soil conductivity model in [4] to be simpler. The potential function of electrical field in the domain of the cross sectional of consideration area is assumed to be a linear of two variables function. The model can simplify to be a hyperbolic partial differential equation. The Lax-Wendroff method in [3] is used to find the approximate solutions within a domain.

In this paper, two mathematical models are used to simulate the salinity in the ground with varied flow velocity. The first is a potential flow model that provides the velocity field of the ground water flow. The second is a dispersion model, where the commonly used governing factor is the two-dimensional dispersion equation that gives the salinity fields. In the simulation process, we used the finite difference method for the both models. Finally, we present a numerical simulation that confirms the results of the technique.

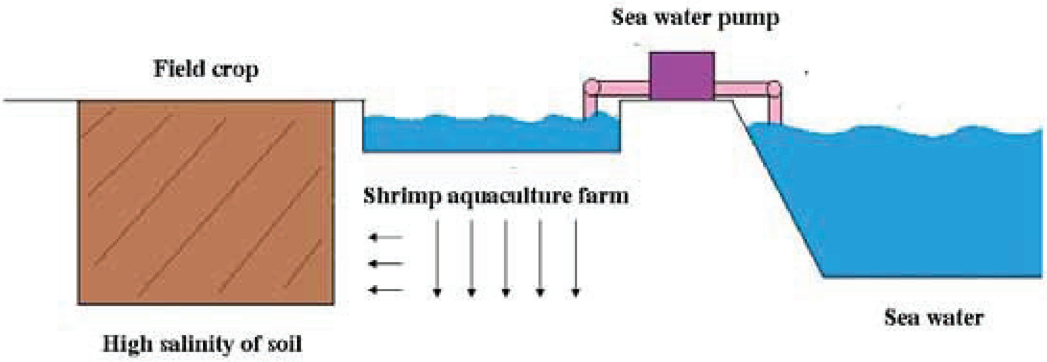

Fig. 1. The saline water from the sea is filled into the shrimp aquaculture farm [7]

2. Mathematical Model

2.1. Potential flow model

Water in marine shrimp farm infiltrate into the ground. They penetrate through pores of the soil and become the groundwater. The soil stratum saturated with the groundwater is called aquifer. The ground water moves through the aquifer as seepage flow. Velocity of the groundwater motion is defined as the rate of volume (m^3/s) divided by the cross section (m^2) consisting of the soil and pores of the soil matrix. The fluid is assumed to be incompressible. Stokes equations apply, because the velocity of the seepage flow is often small.

In the steady state, we are using the D'Arcy law:

$$u = -k\frac{\partial H}{\partial x}, \tag{1}$$

$$v = -k\frac{\partial H}{\partial z}. \tag{2}$$

where k is the hydraulic conductivity of the aquifer, and H is the piezometric head. We now introduce hydraulic potential $\Phi = -kH$. The D'Arcy law becomes

$$u = \frac{\partial \Phi}{\partial x}, \tag{3}$$

$$v = \frac{\partial \Phi}{\partial z}. \tag{4}$$

Since the groundwater is incompressible, the combining these with the equation of continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0, \tag{5}$$

we obtain the Laplace equation,

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2}. \tag{6}$$

Thus we can treat the groundwater flow as a potential flow with following conditions:

$$\frac{\partial \Phi}{\partial n}(0, z) = 0, \tag{7}$$

$$\frac{\partial \Phi}{\partial n}(100, z) = 0, \tag{8}$$

where $z \in [0,50]$, and

$$\frac{\partial \Phi}{\partial n}(x,50) = 0, \tag{9}$$

where $x \in [0,100]$, and

$$\Phi(x) = \begin{cases} 0 & ,0 \le x \le 40 \\ 3.8\times10^{-6} & ,45 \le x \le 100 \end{cases}, \tag{10}$$

$$\frac{\partial \Phi}{\partial n}(x,0) = 0, \tag{11}$$

where $x \in (40,45)$.

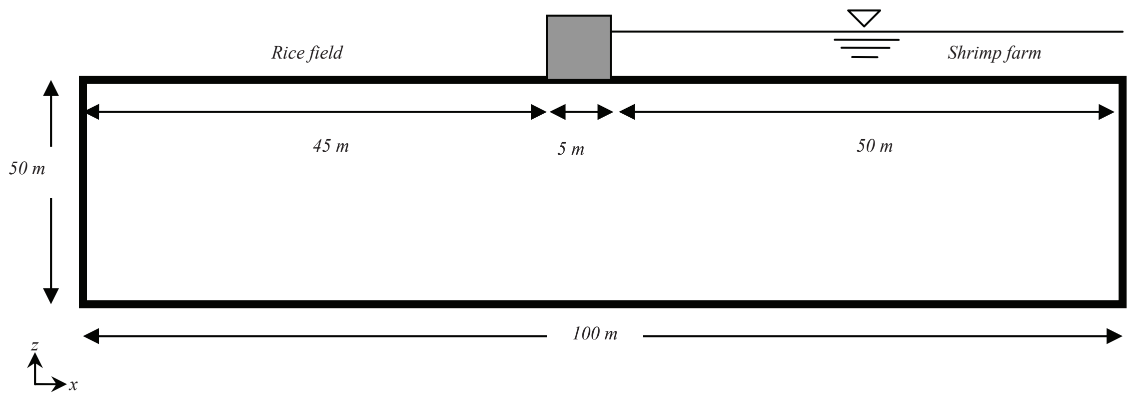

Fig. 2. Groundwater flow under shrimp farm and rice field

2.2. Dispersion model

In a soil salinity model, the governing equations are the dynamic two-dimensional advection-dispersion equations. Since the diffusivities of salinity in soil in *x-z* directions are similar, the diffusion coefficients of diffusion terms in both directions on dispersion equation are equal. A simplified representation by averaging the equation over the surface

$$D\frac{\partial^2 C}{\partial x^2}+D\frac{\partial^2 C}{\partial z^2}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial z}=0\,, \tag{12}$$

where *C(x,z)* is the salinity averaged in the point *x* and depth at *z* and, and *u(x,z)* is the velocity in x-direction and v(x,z) is the velocity in z-direction. The boundary conditions as follows.

$$\frac{\partial C}{\partial n}(0,z)=0,\ \text{where}\ z\in[0,50] \tag{13}$$

$$\frac{\partial C}{\partial n}(100,z)=0\,,\qquad \text{where}\ z\in[0,50]\,, \tag{14}$$

$$\frac{\partial C}{\partial n}(x,50)=0,\ \text{where}\ x\in[0,100]\,, \tag{15}$$

$$C(x,0)=\begin{cases}35 & ,0\le x<40\\ 0 & ,40\le x\le 45\end{cases}, \tag{16}$$

$$\frac{\partial C}{\partial n}=0,\ \text{where}\ 45<x\le 100\,. \tag{17}$$

3. Numerical Experiments

The potential flow model provides the velocity field the groundwater. Then the calculated results of the model will be input into the dispersion model which provides the salinity concentration field.

3.1. A finite difference method for the potential flow model

The potential equation is transformed into non-dimensional form of equation. We now discretise Eq.(6) by dividing the interval [0,1] in x-direction into M subintervals such that $M\Delta x=1$ and the interval $[0,0.5]$ into N subintervals such that $N\Delta Z=0.5$. We can then approximate $\Phi(x_m,z_n)$ by Φ_m^n, value of the difference approximation of $\Phi(x,z)$ at point $X=m\Delta X$ and $Z=n\Delta Z$, where $0\le m\le M-1$ and $0\le n\le N-1$ which M and N are positive integers. Using the central difference method [3] on Eq.(6), we can obtain

$$\frac{\partial^2\Phi}{\partial x^2}\approx\frac{\Phi(x-h,z)-2\Phi(x,z)+\Phi(x+h,z)}{h^2}\,, \tag{18}$$

$$\frac{\partial^2\Phi}{\partial z^2}\approx\frac{\Phi(x,z-k)-2\Phi(x,z)+\Phi(x,z+k)}{k^2}\,. \tag{19}$$

Since *h*=*k*, we can obtain

$$\Phi_m^{n+1}=\Phi_{m-1}^n+\Phi_m^n+\Phi_{m+1}^n+\Phi_m^{n-1}\,. \tag{20}$$

3.2. A finite difference method for the dispersion model

The advection-dispersion equation is transformed into non-dimensional form of equation. We now discretise Eq.(12) by dividing the interval [0,1] in x-direction into M subintervals such that $M\Delta x = 1$ and the interval $[0,0.5]$ into N subintervals such that $N\Delta Z = 0.5$. We can then approximate $C(x_m, z_n)$ by C_m^n, value of the difference approximation of $C(x,z)$ at point $X = m\Delta X$ and $Z = n\Delta Z$, where $0 \le m \le M-1$ and $0 \le n \le N-1$ which M and N are positive integers. Using the central difference method [3] on Eq.(6), we can obtain

$$\frac{\partial^2 C}{\partial x^2} \approx \frac{C(x-h,z)-2C(x,z)+C(x+h,z)}{h^2}, \tag{21}$$

$$\frac{\partial^2 C}{\partial z^2} \approx \frac{C(x,z-k)-2C(x,z)+C(x,z+k)}{k^2}, \tag{22}$$

$$\frac{\partial C}{\partial x} \approx \frac{C(x+h,z)-C(x-h,z)}{2h}, \tag{23}$$

$$\frac{\partial C}{\partial z} \approx \frac{C(x,z+k)-C(x,z-k)}{2k}. \tag{24}$$

Since h=k, we can obtain

$$(1+r)C_m^{n+1} = (r-1)C_m^{n-1} - (1-p)C_{m-1}^n + 4C_m^n - (1+p)C_{m+1}^n. \tag{25}$$

4. Numerical Results

We consider the cross sectional area of a rice filed near marine shrimp aquaculture farm width 100 m and depth is 50 m. The area is meshed by 1,250 grids points with grid space is 2 m. The boundary conditions of the area is specified Eqs.(7)-(11) and Eqs.(13)-(17). The velocity of groundwater flow and salinity of soil are shown on Table 1 and Figures 3-5 respectively.

Table 1. The concentration of salinity in the soil

z,x	0	10	20	30	40	50	60	70	80	90
0	35.0000	35.0000	35.0000	35.0000	0.0000	15.3788	16.9918	16.5819	15.2722	12.6096
5	31.7469	31.5347	30.5941	27.8664	21.1328	17.8271	17.5432	16.8913	15.7012	13.9188
10	28.9720	28.6927	27.6073	25.4227	22.3413	19.9423	18.6602	17.7162	16.7581	15.8478
15	26.8589	26.6159	25.7492	24.2754	22.4543	20.7963	19.5528	18.6082	17.8411	17.2858
20	25.3449	25.1627	24.5386	23.5423	22.3415	21.1630	20.1565	19.3507	18.7375	18.3430
25	24.2898	24.1591	23.7183	23.0271	22.1906	21.3336	20.5550	19.9084	19.4216	19.1207
30	23.5667	23.4726	23.1565	22.6612	22.0552	21.4173	20.8175	20.3069	19.9204	19.6836
35	23.0831	23.0132	22.7779	22.4079	21.9501	21.4593	20.9880	20.5799	20.2684	20.0773
40	22.7775	22.7225	22.5369	22.2436	21.8776	21.4803	21.0937	20.7550	20.4945	20.3341
45	22.6120	22.5648	22.4056	22.1531	21.8363	21.4899	21.1502	20.8506	20.6190	20.4760
50	22.5663	22.5213	22.3694	22.1280	21.8246	21.4923	21.1657	20.8771	20.6536	20.5155

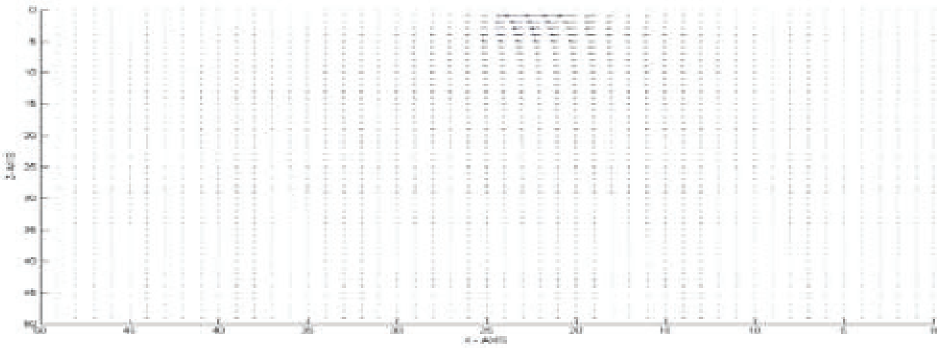

Fig. 3. The velocity fields of groundwater flow

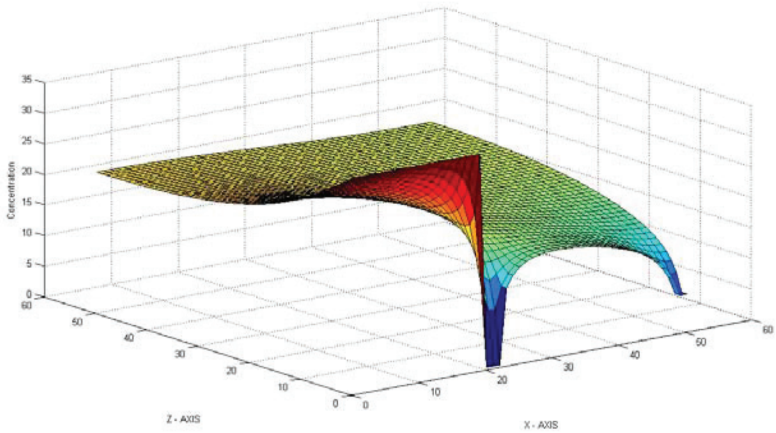

Fig. 4. The surface plot of salinity concentration level in soil

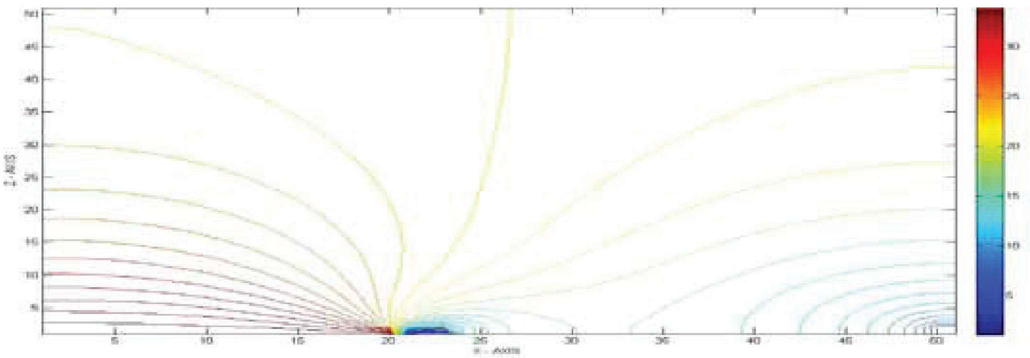

Fig. 5. The contour plot of salinity concentration level in soil

5. Discussion and Conclusion

By the calculated solutions, we can see that the positions near a shrimp farms in the consideration area have higher salinity more than far away positions. The deeper positions also have the soil salinity greater than the surface. We can obtain their approximate solutions on the surface have salinity under the standard. These are then the study area is possible to produce the crop. For the real-world problem, the real data for the assuming the boundary conditions must amend to be more suitable for each terrains.

Acknowledgements

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